Dimensional crossover and driven interfaces in disordered ferromagnets

L. Roters* and K. D. Usadel†
Theoretische Tieftemperaturphysik, Gerhard-Mercator-Universität Duisburg, 47048 Duisburg, Germany

(Received 28 August 2001; published 3 January 2002)

We study the depinning transition of driven interfaces in thin ferromagnetic films driven by external magnetic fields. Approaching the transition point the correlation length increases with decreasing driving. If the correlation length becomes of the order of the film thickness a crossover to two-dimensional behavior occurs. From the corresponding scaling analysis, we determine the exponents characterizing the transition of the three-dimensional system.

DOI: 10.1103/PhysRevE.65.027101 PACS number(s): 68.35.Rh, 75.10.Hk, 75.40.Mg

I. INTRODUCTION

We study interface motion in the random-field Ising model (RFIM). In this model an interface separates regions of opposite spin orientation. A magnetic field forces the interface to move whereas this motion is hindered by the disorder. At zero temperature, a permanent interface motion is found if the driving field H exceeds a critical threshold $H_{\rm c}$ determined by the disorder. The vanishing of the interface velocity at $(H=H_{\rm c}|T=0)$ is, in general, referred to as a continuous phase transition [1–3]. Considering three-dimensional systems, for instance, the interface velocity v vanishes at the so-called pinning/depinning transition according to $v(T=0)\sim(H-H_{\rm c})^{\beta_{3d}}$ and $v(H=H_{\rm c})\sim T^{1/\delta_{3d}}$, respectively (see, e.g., [3,4]). The correlation length diverges algebraically: $\xi\sim(H-H_{\rm c})^{-\nu_{3d}}$ [3,5].

This pinning/depinning transition can be understood within the framework of the renormalization group theory (see, e.g., Refs. [6–10] and references therein). Within this theory it is found that the order parameter of a continuous phase transition is a generalized homogenous function of, in general, several thermodynamic parameters [6]. In many cases temperature and magnetic field belong to these parameters. Additional parameters may cause crossovers. Examples are spatial anisotropies in Heisenberg models [11], dipolar effects [11], or restrictions due to geometry like in thin films (see, e.g., Ref. [12] and references therein). In the case of thin films the system behaves like a three-dimensional system only as long as the correlation length ξ is small as compared to the film thickness l. Approaching the transition point the component ξ_l of the correlation length perpendicular to the film layers is bounded by the film thickness. If ξ_l/l becomes of the order of unity a crossover from three- to two-dimensional behavior occurs. Experiments that determine domain wall velocities in magnetic films typically image the magnetic state of a sample by looking *onto* the sample using the magneto-optical polar Kerr effect and a charge coupled device (CCD) camera [13,14]. If the film is sufficiently thin it is possible to obtain the interface position from snapshots generated by the CCD camera and to calculate the interface velocity from the time dependence of this position. This was done in Ref. [14], where a Pt(3.4 nm)/Co(0.5 nm)/Pt(6.5 nm) film with perpendicular anisotropy was investigated. Due to the film thickness the authors saw evidence to neglect the height dependence of the interface position and to treat the interface not as a two-dimensional interface but as a one-dimensional line. With increasing film thickness, however, this assumption fails because then the correlation length drops below the film thickness causing a crossover to three-dimensional behavior. This scenario is investigated in the present paper in the context of driven interfaces in the random field Ising model.

II. SIMULATION

The RFIM is defined by the Hamiltonian

$$\mathcal{H}=-\frac{J}{2}\sum_{\langle i,j\rangle}S_iS_j-H\sum_i S_i-\sum_i h_iS_i. \tag{1}$$

The first term is the exchange interaction of neighboring spins and the sum is taken over all pairs of them $(S_i=\pm1)$. The second term specifies the coupling to the driving field H. Additionally, the spins are coupled to independent quenched local random-fields h_i characterized by their probability density $p(h_i)$ given by

$$p(h_i)=\begin{cases}(2\Delta)^{-1} & \text{for } |h_i|<\Delta\\ 0 & \text{otherwise.}\end{cases} \tag{2}$$

We use a random-sequential update with transition probabilities according to a heat bath algorithm in the limit of zero temperature. Since in the vicinity of the critical point finite-size effects may not only occur due to the finite film thickness but also due to the other finite extensions of our system, we calculated the interface velocities for each film thickness l varying the other extensions of the system. For sufficient large extensions, we observed no finite-size effects from which we concluded that the data presented in the following correspond within negligible errors to those of an infinite extended film of thickness l. Note that for this analysis the extensions of the system must be increased if one approaches the critical point. But this increase requires an increase in computer time that restricts, therefore, the data to values not

*Email address: lars@thp.uni-duisburg.de
†Email address: usadel@thp.uni-duisburg.de

 ©2002 The American Physical Society

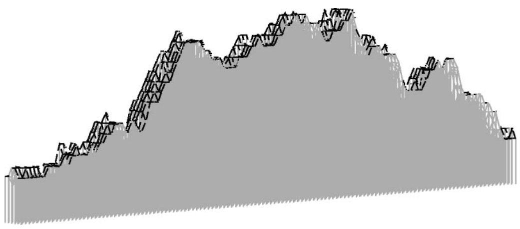

FIG. 1. Snapshot of a moving interface obtained in a film consisting of six magnetic layers. Overhangs, which are rare and small, are not shown. In the gray sketched area below the interface the spins are aligned parallel to the driving field. Spins above the interface are aligned antiparallel and not shown.

too close to the critical point. We investigate system sizes of up to $l \times L^2 = 8 \times 1024^2$ unit cells of a body centered cubic lattice.

We apply periodic boundary conditions in the directions parallel to the film and antiperiodic ones perpendicular to the interface (see Ref. [3]). The interface moves along the [100] direction of the bcc lattice resulting in a finite interface velocity for any driving field $H \neq 0$ in the absence of disorder. The same behavior is found, for instance, on simple cubic lattices with an interface moving along the diagonal direction of the lattice [2,3] and on diamond lattices with the interface moving along the [100] direction. The schematic phase diagram in Ref. [3] applies to all of these cases. In particular, a continuous phase transition is found for $\Delta > J$ as long as nucleation does not occur. This is the case for $\Delta = 3$, which turns out to be a convincing choice because then the dimensional crossover is numerically accessible for a broad range of system sizes.

III. RESULTS

In our simulations we start with an originally flat interface that is built into the system. After a transient regime the interface reaches a stationary state. In Fig. 1, we show the snapshot of an interface configuration (for $H = 1.8$ and $\Delta = 3.0$). Overhangs are not displayed since they are rare and small. We obtain the interface velocity from the mean interface position $\overline{h}$ at a given time t according to $v = \partial \overline{h} / \partial t$. In Ref. [3] it was argued that this velocity is a generalized homogeneous function of temperature and driving field. We generalize this ansatz to the present situation and include additionally a thickness dependence of v,

$$v = \lambda v(\lambda^{-1/\beta_{3d}} \eta, \lambda^{-\delta_{3d}} T, \lambda^{-a_l} l^{-1}). \tag{3}$$

Here, T denotes the temperature and $\eta = H - H_c^{3d}$ the reduced driving field. In a bulk system we recover under the assumption $v(l^{-1} = 0) = \text{finite}$ (3) $v \sim T^{1/\delta_{3d}} f(\eta / T^{1/\beta_{3d}\delta_{3d}})$ that is known to be satisfied in the RFIM [2,3]. In the following we consider only $T = 0$. Choosing $\lambda = l^{-\beta_{3d}/\nu_{3d}}$ in Eq. (3) we obtain for the velocity the following scaling behavior:

$$v = l^{-\beta_{3d}/\nu_{3d}} f(\eta l^{1/\nu_{3d}}) \tag{4}$$

with $f(x \to \infty) \sim x^{\beta_{3d}}$. In this limit the value of H_c coincides with that of the bulk system. For any finite l, however, H_c becomes l dependent and is shifted according to $H_c(l) - H_c^{3d} \approx x^* l^{-1/\nu_{3d}}$ (see Ref. [12] and references therein). Inserting this relation into Eq. (4) one finds that $f(x)$ does not vanish at $x = 0$ but at a finite value x^*.

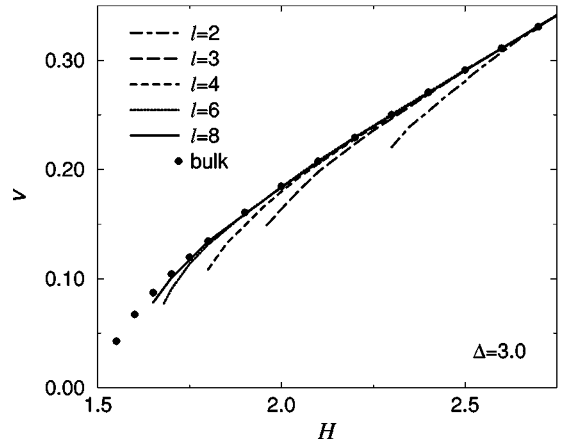

FIG. 2. Interface velocities v obtained for various film thicknesses l and different driving fields H. We also plot the interface velocity of the bulk system. Approaching $H_c(l)$ the system size must be increased in order to find negligible finite-size effects (see Sec. II). The curves terminate at fields, where the finite-size effects set in for the largest set sites accessible numerically.

In Fig. 2 we plot $v(H)$ for different film thicknesses. For a comparison, we also plot velocities obtained in a three-dimensional system. From the data it is evident that the curves $v(H)$ for different film thicknesses deviate more and more from the bulk behavior with decreasing H. In the corresponding region of H values the crossover from two- to three-dimensional behavior occurs, which thus is numerically accessible. In the crossover region the interface velocity in a film turns out to be smaller than in the bulk meaning that the threshold field H_c is shifted towards *larger* values of the driving field. Rescaling the velocities according to Eq. (4) yields the data collapse shown in Fig. 3. From the collapse it may be concluded that the scaling function $f(x)$ [see Eq. (4)] vanishes at a value $x^* > 0$, again meaning that the threshold field is shifted towards larger values with decreasing film thickness. We obtain $\beta_{3d} = 0.677 \pm 0.07$ and $H_c^{3d} = 1.491 \pm 0.02$. These values coincide within the error bars with those found in bulk systems. In this case (inset of Fig. 3) we find $\beta^{\text{bulk}} = 0.64 \pm 0.05$ and $H_c^{\text{bulk}} = 1.5 \pm 0.015$ confirming the results of the crossover scaling. From the data collapse in Fig. 3, we also obtain $\nu_{3d} = 0.763 \pm 0.03$. Both β_{3d} and ν_{3d} coincide within the error bars with values found on other lattices and different values of Δ [3–5]. Also, they agree with the exponents of the Edwards-Wilkinson equation with quenched disorder (see, e.g., Refs. [9,10,15] and references therein). For this equation $\beta_{\text{QEW}} \approx 0.62$ and $\nu_{\text{QEW}} \approx 0.77$ was obtained by an ϵ expansion valid to order ϵ^2 [10].

Since the interface velocity satisfies the ansatz Eq. (4), it is possible to draw conclusions about the critical behavior at $H_c(l)$. Considering $f(x)$ with $x = \eta l^{1/\nu_{3d}}$ and taking into account that $\nu_{3d} > 0$ one finds $x = 0$ at $H_c(l)$ for any finite film thickness. On the other hand, a three-dimensional system

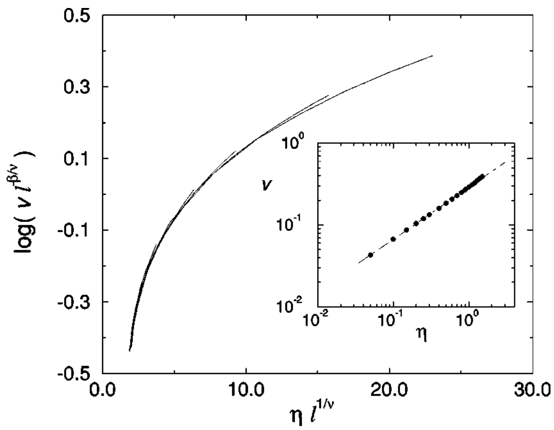

FIG. 3. Scaling plot according to Eq. (4). For convenience we rescale the y axes by using the $\log_{10}$ function. From the data collapse we obtain $\beta_{3d}=0.677\pm0.07$, $\nu_{3d}=0.763\pm0.03$, and $H_c^{3d}=1.491\pm0.02$. The inset shows the interface velocities of the bulk system. We obtain $\beta^{\text{bulk}}=0.64\pm0.05$ and $H_c^{\text{bulk}}=1.5\pm0.015$. For small values of η all curves join the data collapse. For sufficient large η, i.e., well above the crossover region, the scaling ansatz Eq. (4) does not hold as can be seen from the deviations of single curves from the scaling behavior.

corresponds to the limit $l\rightarrow\infty$ and in this case $f(x)$ is determined by the limiting behavior $f(x\rightarrow\infty)\sim x^{\beta_{3d}}$. The critical behavior in a film is, therefore, different as compared to the bulk behavior and the critical exponents β and ν coincide with those of the two-dimensional model for any finite film thickness. In the two-dimensional model $\beta_{2d}\approx0.31$ and $\nu_{2d}\approx1$ was found [2,4]. Taking Eq. (3) into account, we also conclude that the exponent δ characterizing the thermal rounding of the depinning transition $[v\sim T^{1/\delta_{2d}}$ at $H=H_c(l)]$ is given by $\delta_{2d}\approx5$ (see Ref. [2]) for any finite film thickness.

IV. CONCLUSION

In conclusion, we have investigated the depinning transition of a driven interface in thin films. We have found that the critical behavior is governed by the two-dimensional fix point and the corresponding exponents for any finite film thickness. The exponents obtained by the crossover scaling are in agreement with those of the Edwards-Wilkinson universality class. The scaling ansatz used to analyze our data could also be used to determine critical exponents of the depinning transition of three-dimensional systems experimentally, in particular, since present experimental technique use thin films as samples [13,14]. In Ref. [13], for instance, $Co_{28}Pt_{72}$ alloy films were investigated with grain sizes of typically 20 nm and film thicknesses of 5–50 nm. If one naively assumes that by a variation of temperature and/or driving field it is possible to increase the correlation length from the size of the grain to the film thickness, we expect the crossover scaling (3) to work and to yield the exponents of a three-dimensional sample. Note that due to dipolar interactions these exponents need not to coincide with those of the RFIM.

ACKNOWLEDGMENTS

The authors would like to thank S. Lübeck for many useful discussions. This work was supported by the Deutsche Forschungsgemeinschaft via GRK 277 *Struktur und Dynamik heterogener Systeme* (University of Duisburg) and SFB 491 *Magnetische Heteroschichten: Struktur und elektronischer Transport* (Duisburg/Bochum).

[1] A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, England, 1995).
[2] U. Nowak and K. D. Usadel, Europhys. Lett. **44**, 634 (1998).
[3] L. Roters, A. Hucht, S. Lübeck, U. Nowak, and K. D. Usadel, Phys. Rev. E **60**, 5202 (1999).
[4] L. A. N. Amaral, A.-L. Barabási, and H. E. Stanley, Phys. Rev. Lett. **73**, 62 (1994).
[5] H. Ji and M. O. Robbins, Phys. Rev. B **46**, 14 519 (1992).
[6] K. G. Wilson and J. Kogut, Phys. Rep. **12**, 75 (1974).
[7] C. Domb, in *Twentieth Century Physics*, edited by L. M. Brown, A. Pais, and Sir B. Pippard (IOP Publishing Limited/AIP Press Inc., Bristol/New York, 1995), Vol. 1, Chap. 7.2.
[8] L. P. Kadanoff, *Statistical Physics: Statics, Dynamics and Renormalization* (World Scientific Publishing, Singapore, 2000).
[9] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, J. Phys. II **2**, 1483 (1992).
[10] P. Chauve, P. L. Doussal, and K. J. Wiese, Phys. Rev. Lett. **86**, 1785 (2001).
[11] M. E. Fisher, Rev. Mod. Phys. **46**, 597 (1974).
[12] T. W. Capehart and M. E. Fisher, Phys. Rev. B **13**, 5021 (1976).
[13] U. Nowak, J. Heimel, T. Kleinefeld, and D. Weller, Phys. Rev. B **56**, 8143 (1997).
[14] S. Lemerle, J. Ferré, C. Chappert, V. Mathet, T. Giamarchi, and P. Le Doussal, Phys. Rev. Lett. **80**, 849 (1998).
[15] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 882 (1982).